

6-3 Lesson Reading Guide**Tests for Parallelograms****Get Ready for the Lesson**

Read the introduction to Lesson 6-3 in your textbook.

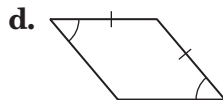
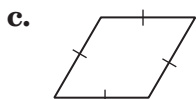
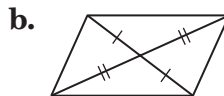
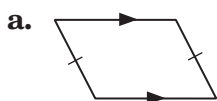
Make two observations about the angles in the roof of the covered bridge.

Read the Lesson

1. Which of the following conditions guarantee that a quadrilateral is a parallelogram?

- A. Two sides are parallel.
- B. Both pairs of opposite sides are congruent.
- C. The diagonals are perpendicular.
- D. A pair of opposite sides is both parallel and congruent.
- E. There are two right angles.
- F. The sum of the measures of the interior angles is 360.
- G. All four sides are congruent.
- H. Both pairs of opposite angles are congruent.
- I. Two angles are acute and the other two angles are obtuse.
- J. The diagonals bisect each other.
- K. The diagonals are congruent.
- L. All four angles are right angles.

2. Determine whether there is enough given information to know that each figure is a parallelogram. If so, state the definition or theorem that justifies your conclusion.

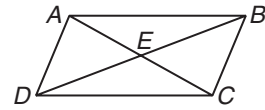
**Remember What You Learned**

3. A good way to remember a large number of mathematical ideas is to think of them in groups. How can you state the conditions as one group about the *sides* of quadrilaterals that guarantee that the quadrilateral is a parallelogram?

6-3 Study Guide and Intervention

Tests for Parallelograms

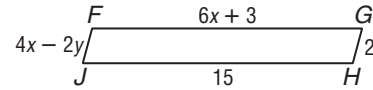
Conditions for a Parallelogram There are many ways to establish that a quadrilateral is a parallelogram.



If:	If:
both pairs of opposite sides are parallel,	$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$,
both pairs of opposite sides are congruent,	$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$,
both pairs of opposite angles are congruent,	$\angle ABC \cong \angle ADC$ and $\angle DAB \cong \angle BCD$,
the diagonals bisect each other,	$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$,
one pair of opposite sides is congruent and parallel,	$\overline{AB} \parallel \overline{CD}$ and $\overline{AB} \cong \overline{CD}$, or $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$,
then: the figure is a parallelogram.	then: $ABCD$ is a parallelogram.

Example Find x and y so that $FGHJ$ is a parallelogram.

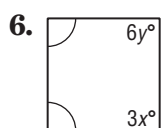
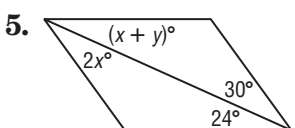
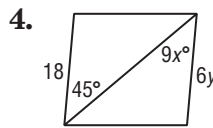
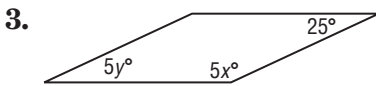
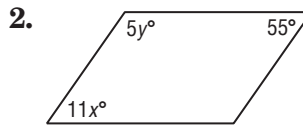
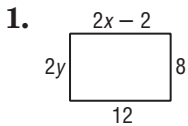
$FGHJ$ is a parallelogram if the lengths of the opposite sides are equal.



$$\begin{aligned}
 6x + 3 &= 15 & 4x - 2y &= 2 \\
 6x &= 12 & 4(2) - 2y &= 2 \\
 x &= 2 & 8 - 2y &= 2 \\
 & & -2y &= -6 \\
 & & y &= 3
 \end{aligned}$$

Exercises

Find x and y so that each quadrilateral is a parallelogram.



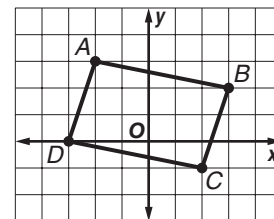
6-3 Study Guide and Intervention *(continued)*

Tests for Parallelograms

Parallelograms on the Coordinate Plane On the coordinate plane, the Distance Formula and the Slope Formula can be used to test if a quadrilateral is a parallelogram.

Example Determine whether $ABCD$ is a parallelogram.

The vertices are $A(-2, 3)$, $B(3, 2)$, $C(2, -1)$, and $D(-3, 0)$.



Method 1: Use the Slope Formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$.

$$\text{slope of } \overline{AD} = \frac{3 - 0}{-2 - (-3)} = \frac{3}{1} = 3 \quad \text{slope of } \overline{BC} = \frac{2 - (-1)}{3 - 2} = \frac{3}{1} = 3$$

$$\text{slope of } \overline{AB} = \frac{2 - 3}{3 - (-2)} = -\frac{1}{5} \quad \text{slope of } \overline{CD} = \frac{-1 - 0}{2 - (-3)} = -\frac{1}{5}$$

Opposite sides have the same slope, so $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$. Both pairs of opposite sides are parallel, so $ABCD$ is a parallelogram.

Method 2: Use the Distance Formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

$$AB = \sqrt{(-2 - 3)^2 + (3 - 2)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26}$$

$$CD = \sqrt{(2 - (-3))^2 + (-1 - 0)^2} = \sqrt{25 + 1} \text{ or } \sqrt{26}$$

$$AD = \sqrt{(-2 - (-3))^2 + (3 - 0)^2} = \sqrt{1 + 9} \text{ or } \sqrt{10}$$

$$BC = \sqrt{(3 - 2)^2 + (2 - (-1))^2} = \sqrt{1 + 9} \text{ or } \sqrt{10}$$

Both pairs of opposite sides have the same length, so $ABCD$ is a parallelogram.

Exercises

Determine whether a figure with the given vertices is a parallelogram. Use the method indicated.

1. $A(0, 0)$, $B(1, 3)$, $C(5, 3)$, $D(4, 0)$;
Slope Formula

2. $D(-1, 1)$, $E(2, 4)$, $F(6, 4)$, $G(3, 1)$;
Slope Formula

3. $R(-1, 0)$, $S(3, 0)$, $T(2, -3)$, $U(-3, -2)$;
Distance Formula

4. $A(-3, 2)$, $B(-1, 4)$, $C(2, 1)$, $D(0, -1)$;
Distance and Slope Formulas

5. $S(-2, 4)$, $T(-1, -1)$, $U(3, -4)$, $V(2, 1)$;
Distance and Slope Formulas

6. $F(3, 3)$, $G(1, 2)$, $H(-3, 1)$, $I(-1, 4)$;
Midpoint Formula

7. A parallelogram has vertices $R(-2, -1)$, $S(2, 1)$, and $T(0, -3)$. Find all possible coordinates for the fourth vertex.